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Flow, Random Arrivals, and Queuing

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Objectives

- Understand the effects of variation on throughput
- Understand the effects of scheduled and non-scheduled arrival patterns on flow
- Understand basic queuing theory and calculations

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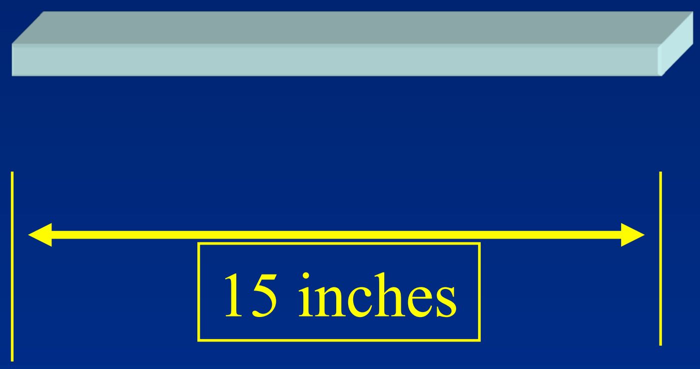
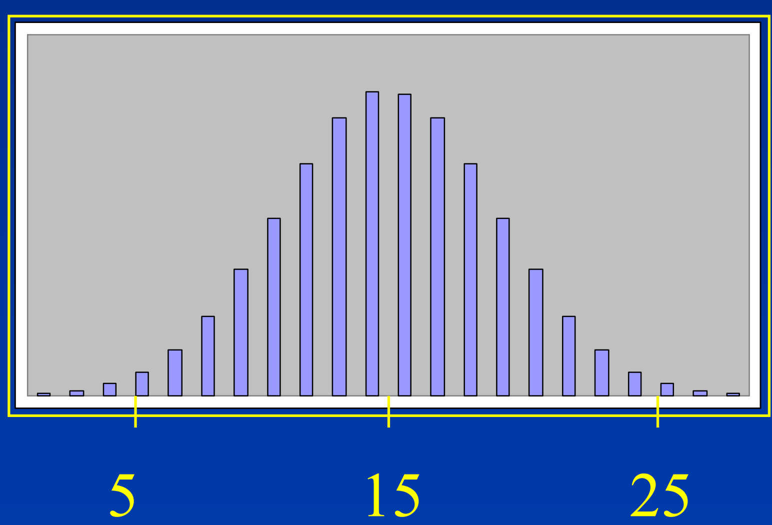
Variation

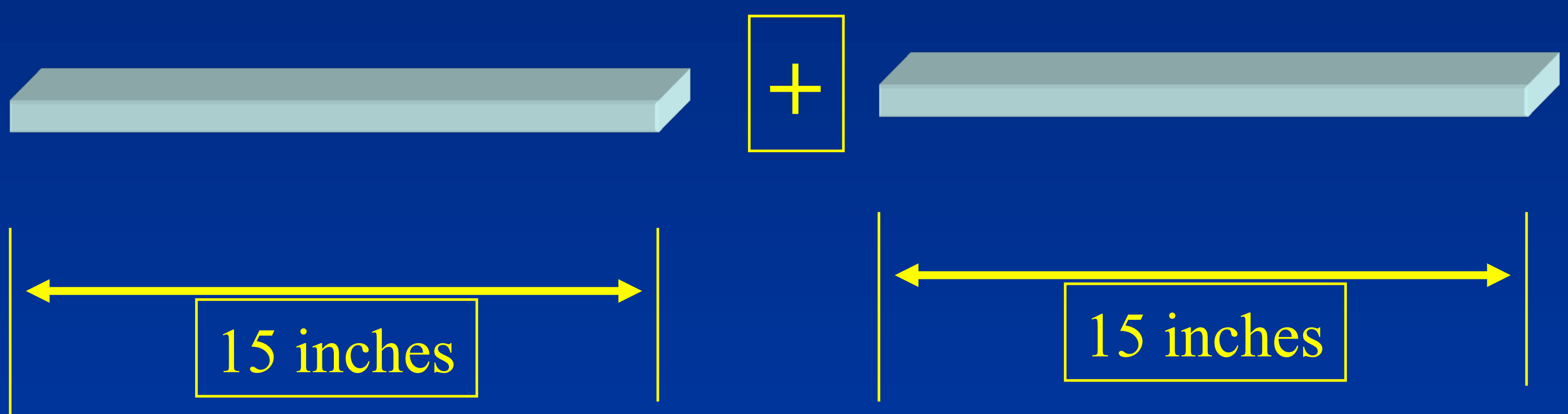


Typical expression of variation for a <u>very bad</u> carpenter might be as follows:



Lengths are *normally* distributed with a *mean* of 15 inches and a *standard deviation* of 5 inches.

What happens when you try to add the 2 pieces?



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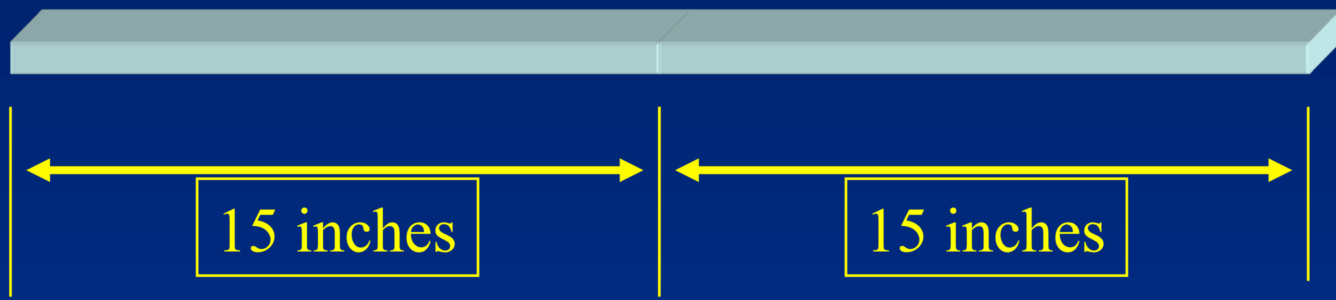
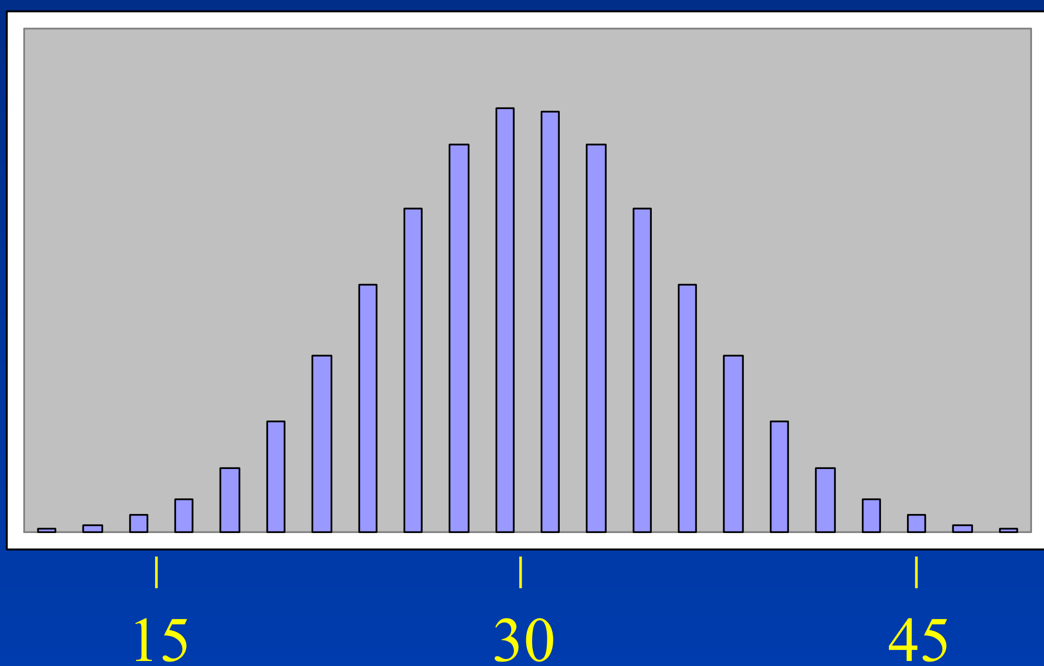
The Variation is Additive





Mean = 30
Standard Deviation = 7.071

Note: Variances are additive, not S.D.

SQRT(25 +25)= 7.071

Clinic Example

- Day Clinic - General medical care by appointment only during the day (8am - 4pm).
- Evening Clinic - Walk-in clinic operated during the evening (4pm-midnight). No appointments accepted
- Both are staffed by a single physician.

σ Sigma Medical

- Day Clinic - general medical care by appointment only during the day (8am - 4pm).
- Evening Clinic - immediate care walk-in clinic operated during the evening (4pm-midnight).
- Both are staffed by a single physician.

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Evening Clinic - Assumptions

1. The physician is always *on time* for the start of his shift.
2. On average, 4 patients arrive per hour.

   Assume 1 patient arrives *every* 15 minutes.
3. The time it takes the physician to treat a patient averages 12 minutes (can treat 5 per hour).

   Assume *exactly* 12 minutes per patient.

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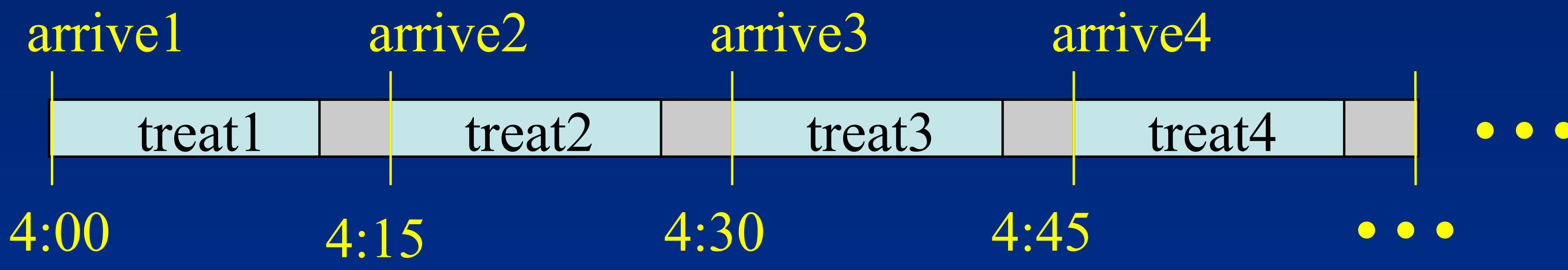
Evening Clinic - Capacity & Performance



- Daily number of patients? 32
- Average patient wait time? 0
- Percent of patients who experience wait? 0
- Percent of time the physician is idle? 20

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Evening Clinic - Assumptions

1. The physician is always *on time* for the start of his shift.
2. On average, 4 patients arrive per hour.

   Assume ~~1 patient arrives *every* 15 minutes.~~ **Poisson arrival process.**

3. The time it takes the physician to treat a patient averages 12 minutes (can treat 5 per hour).

   Assume *exactly* 12 minutes per patient.

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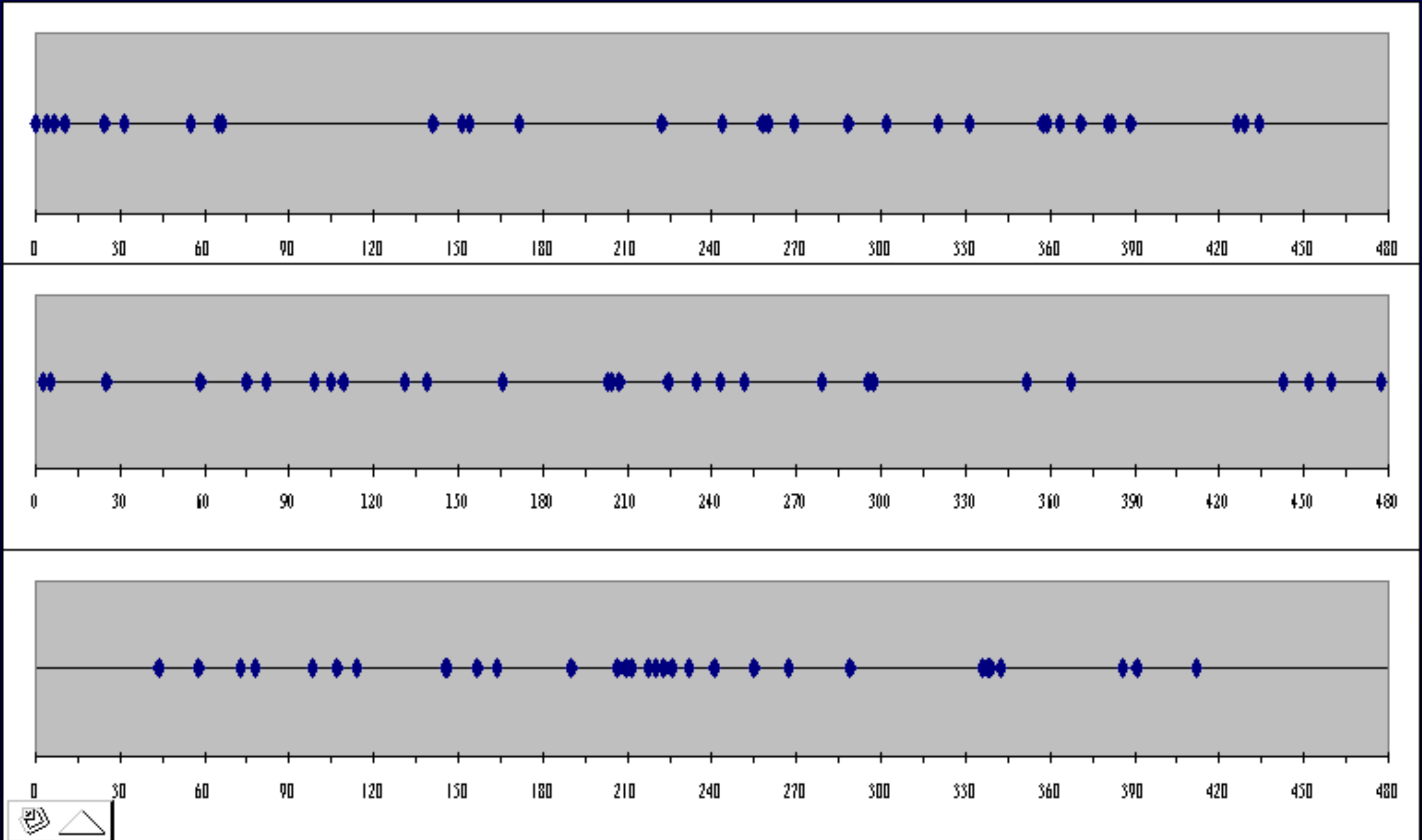
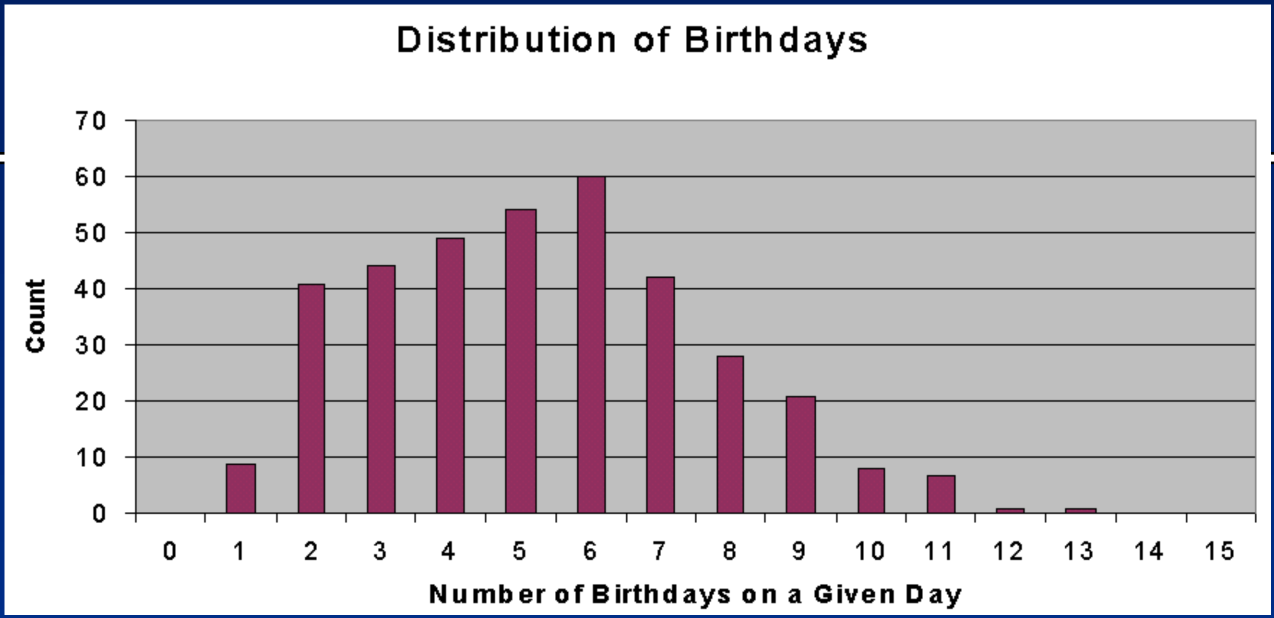
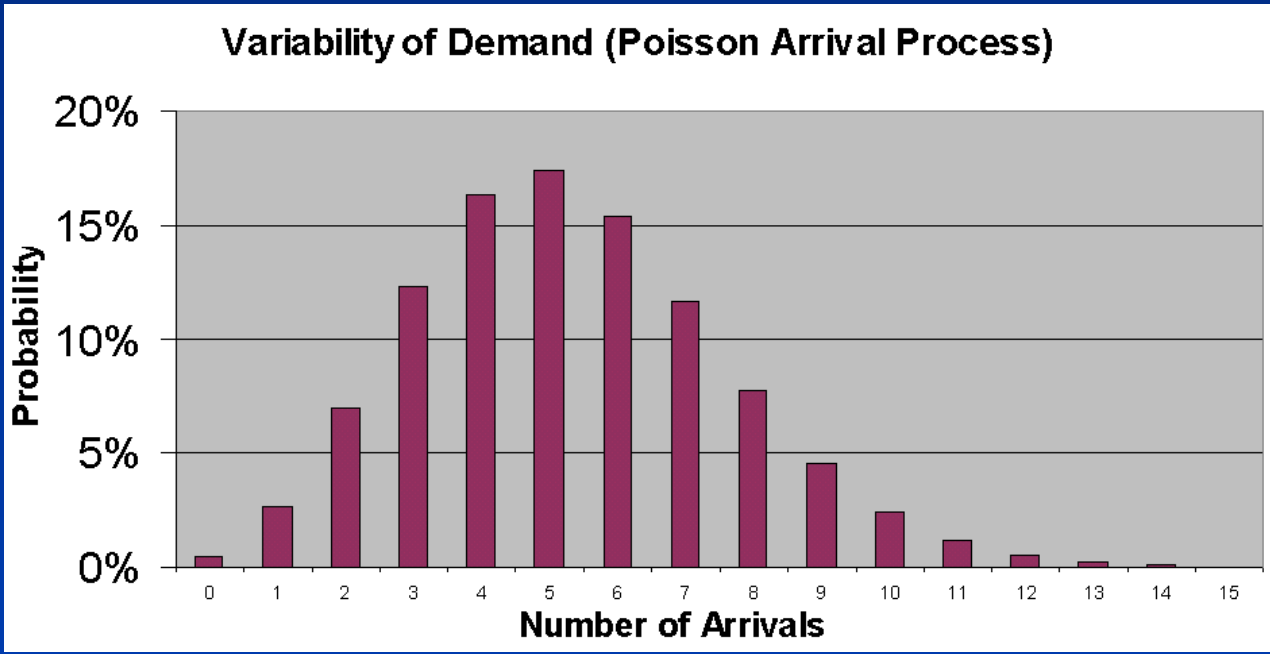
Evening Clinic - Poisson Arrivals





From birthdate information for 1,938 patients



From Poisson expression using average arrival rate of 5

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Evening Clinic - Assumptions

1. The physician is always *on time* for the start of his shift.
2. On average, 4 patients arrive per hour.

   Assume ~~1 patient arrives *every* 15 minutes.~~ **Poisson arrival process.**
3. The time it takes the physician to treat a patient averages 12 minutes (can treat 5 per hour) **service times exponentially distributed.**

   Assume ~~*exactly* 12 minutes per patient.~~

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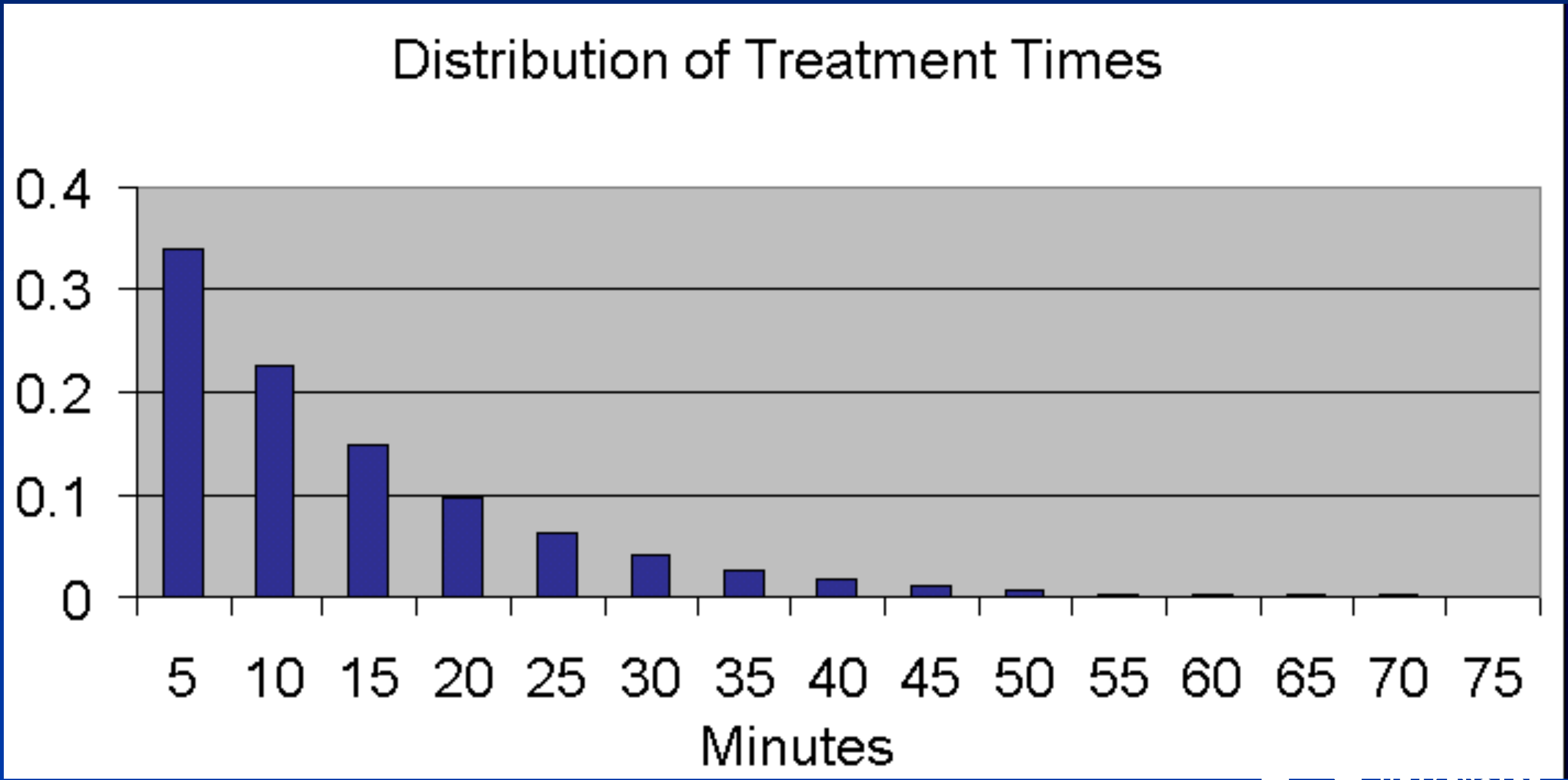
Exponential Distribution of Treatment Times



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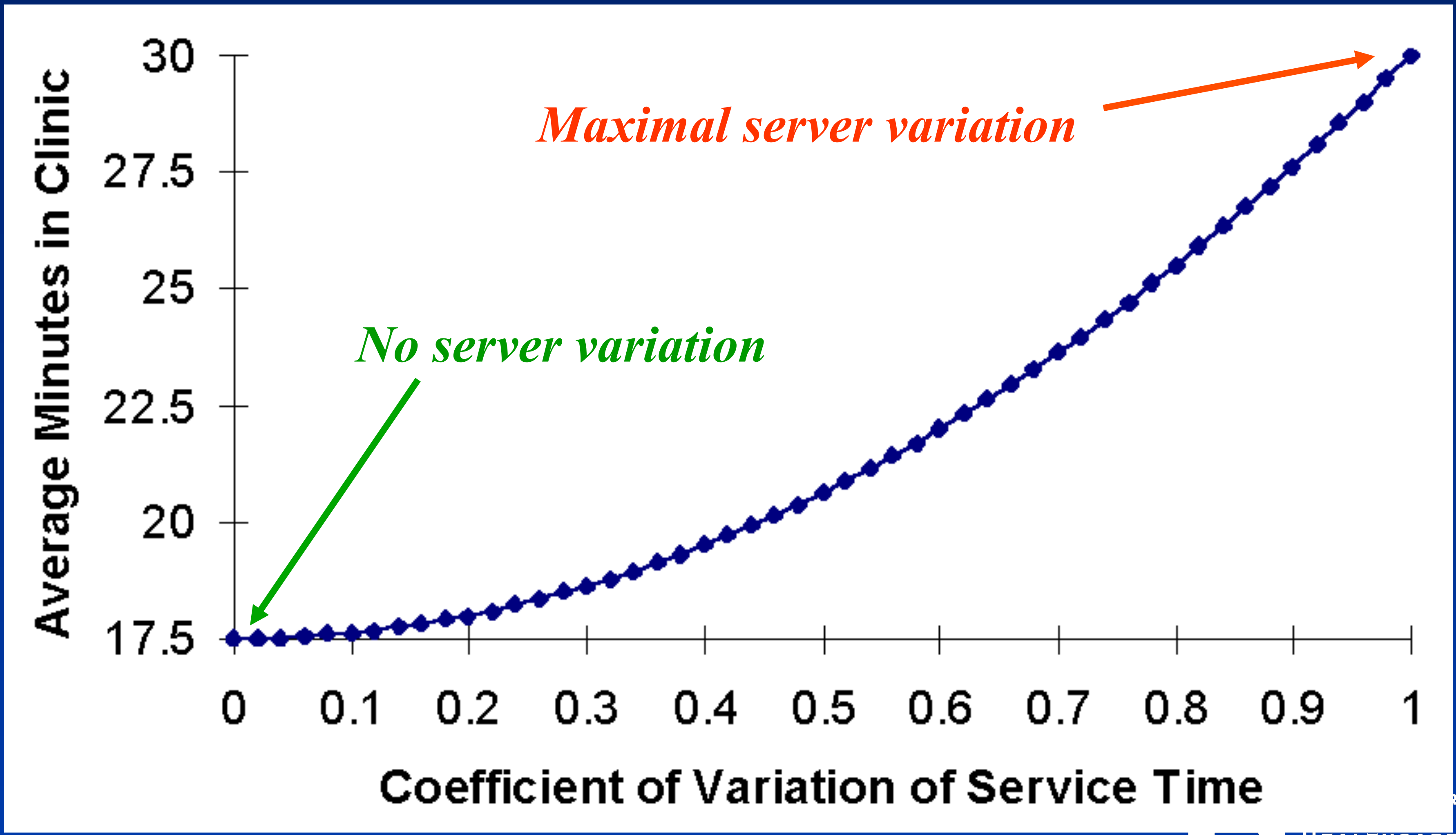
Evening Clinic - Waiting Times

- Average patient wait time? _30__
- Percent of patients who experience ...
  - any wait? __70__
  - more than 10 minutes of wait? __55__
  - more than 30 minutes of wait? __30__
  - more than 60 minutes of wait? __12__
- Percent of time the physician is idle? _20__
- Average number of patients waiting? _3__

Evening Clinic – Take Aways

- Unscheduled arrivals tend to follow a Poisson process and impose considerable load variation on a system.

Arrival Rate of 10/hour, Service Rate of 12/hour, Server Utilization of 83.33%



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Evening Clinic – Take Aways

- Unscheduled arrivals tend to follow a Poisson process and impose considerable load variation on a system.
- Even systems with built-in “capacity cushions” can exhibit extremely long waits.

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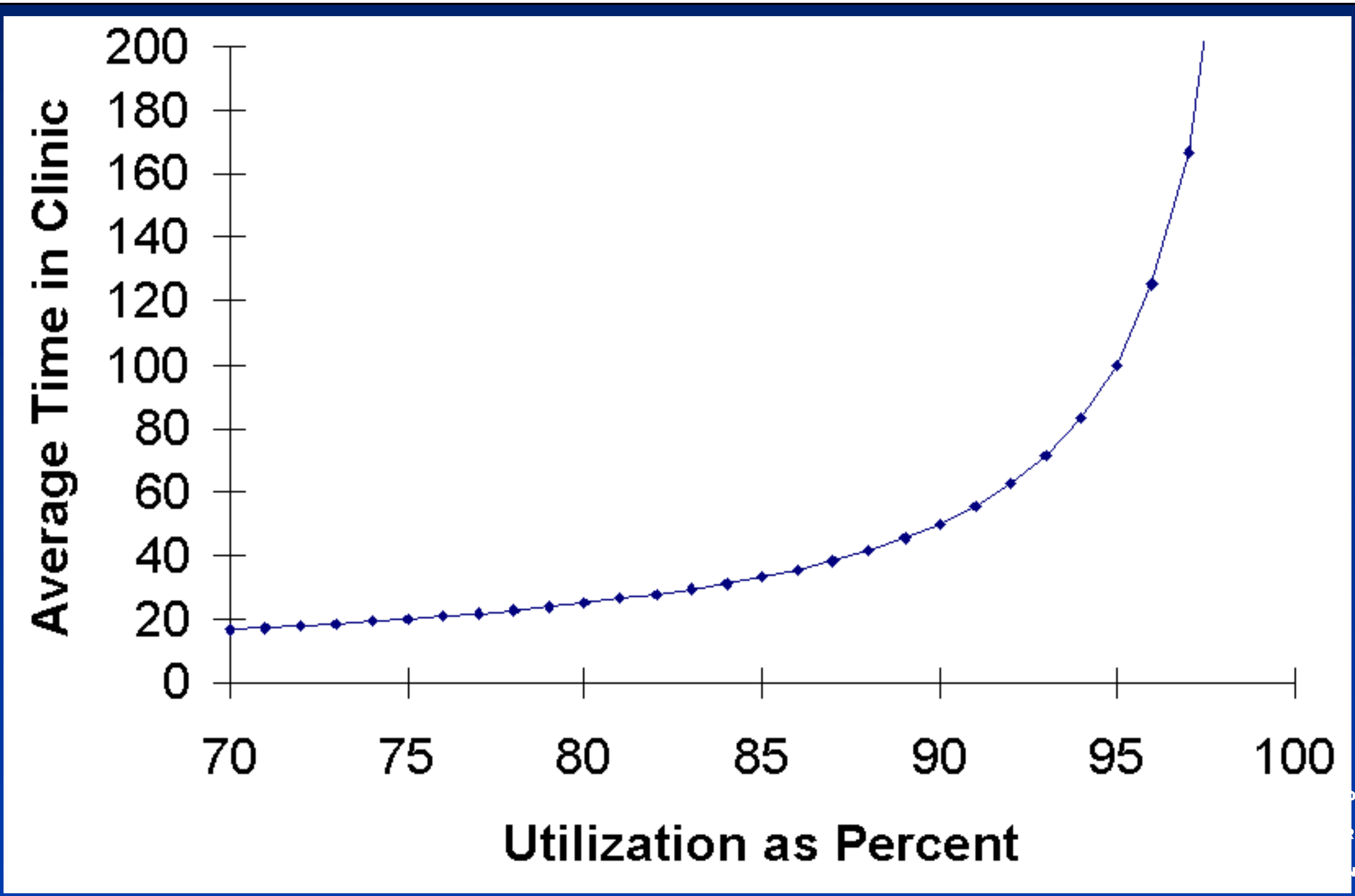
Queue Behavior as a Function of Server Utilization



Evening Clinic – Take Aways

- Unscheduled arrivals tend to follow a Poisson process and impose considerable load variation on a system.
- Even systems with built-in “capacity cushions” can exhibit extremely long waits.
- Customer experiences in a walk-in system are considerably more varied than in a scheduled system.
- For either of the two service systems, a calculation of “capacity” is not easy.

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Take Aways

- Identify sources of variation and try to reduce the ones that you can.
- Use information to reduce variation.
- Adjust utilization expectations according to demand variation and tolerance for waiting.
- Look for pooling opportunities but keep in mind that random demands result in random amounts of idle time.

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A Basic Queue

Server

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A Basic Queue

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A Basic Queue

A Basic Queue

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A Basic Queue

Queuing Analysis

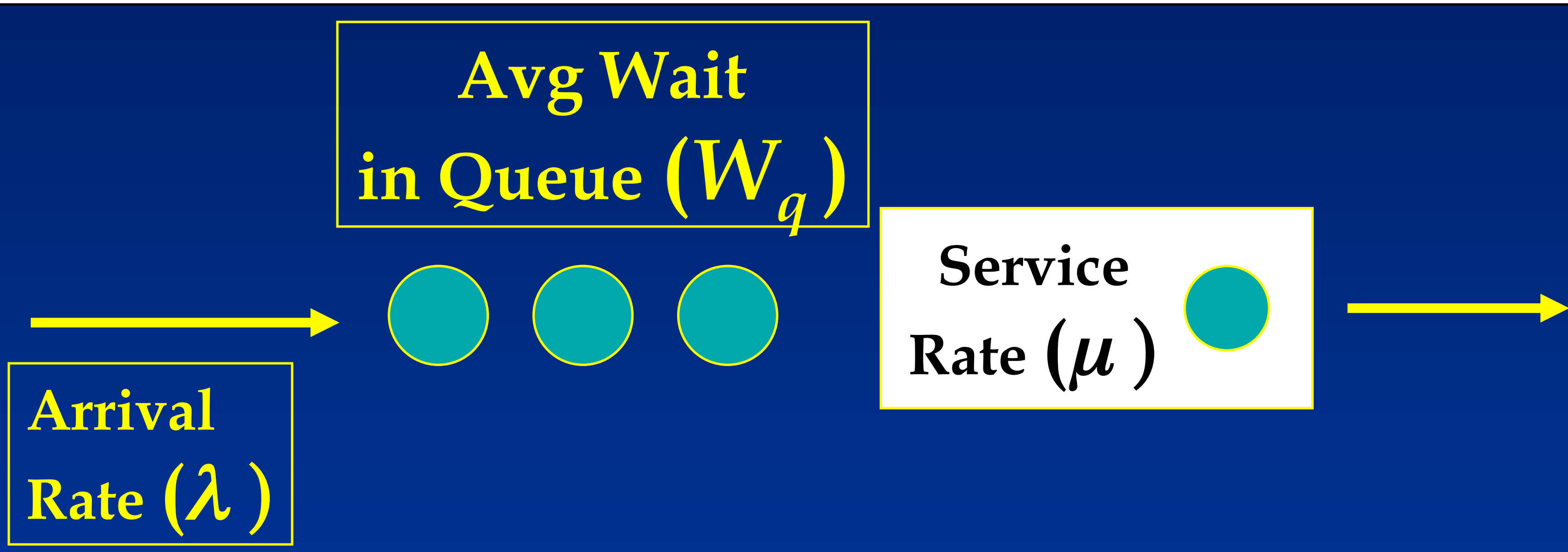
Service Rate (μ)

Queuing Analysis

Service Rate (μ)

Arrival Rate (λ)

Queuing Analysis



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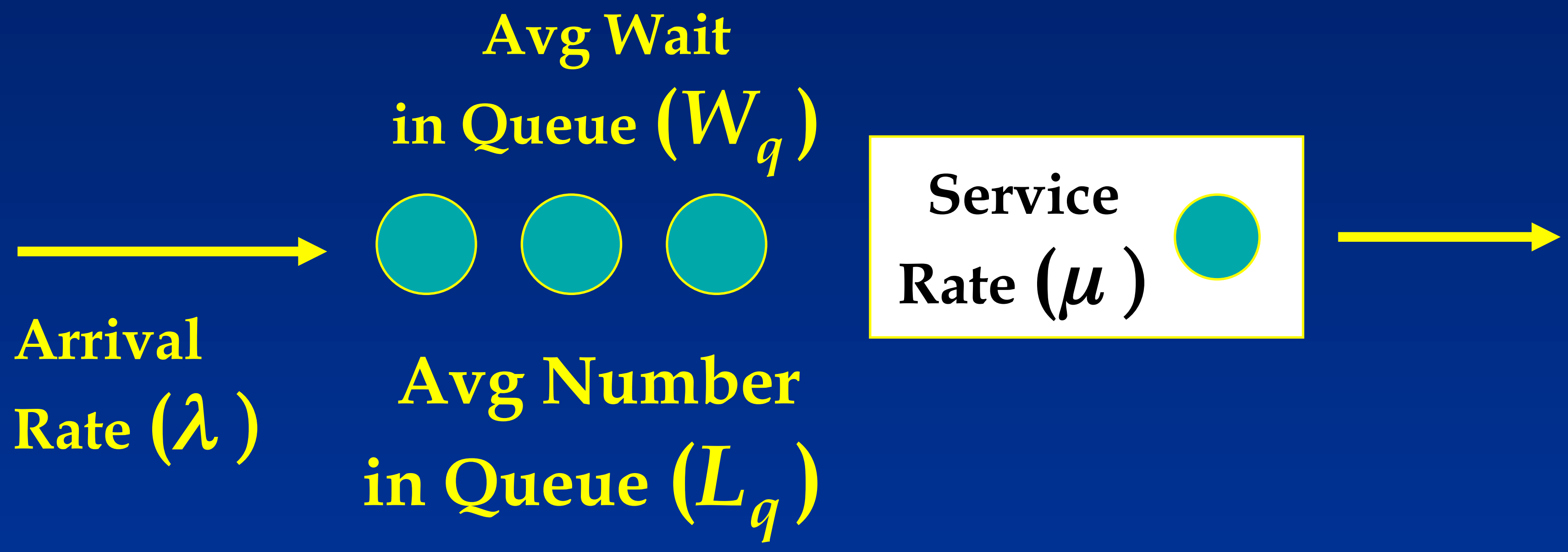
Queuing Analysis



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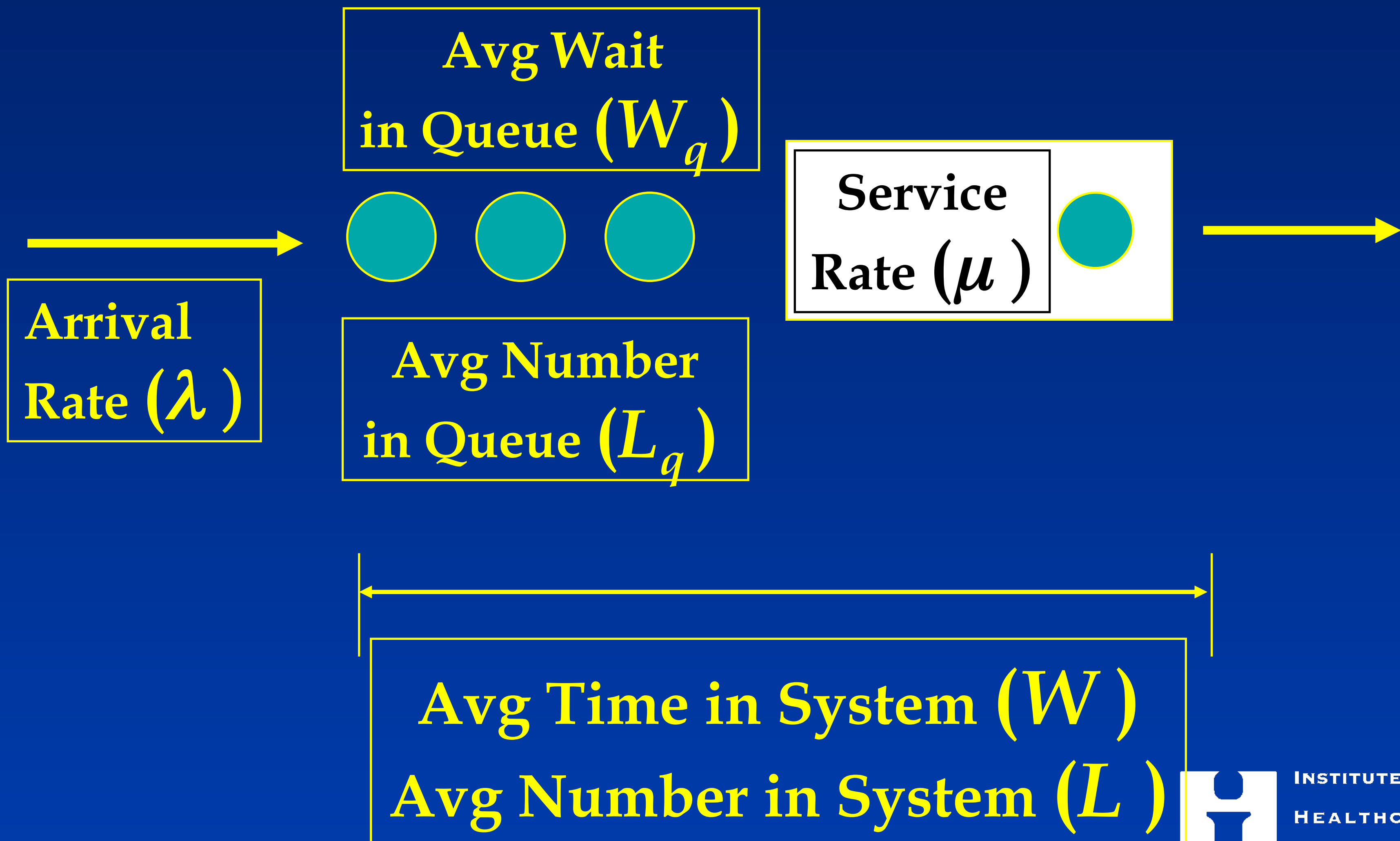
Queuing Analysis



Queuing Basics

For a single server system with Poisson arrivals (of rate λ) and Exponential distribution of service times (of rate μ), we can calculate steady-state estimates of:

1. Utilization (ρ)
2. Average Time in the System (W)
3. Average Number of People in the System (L)
4. Average Wait Time (Wq)
5. Average Line Length (Lq)

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1. Utilization (ρ)

Utilization $\rho = \lambda / \mu$

For the example with λ=4/hour and μ=5/hour,

ρ = 4/5 = .80 or 80% utilization of physician.

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2. Average Time-in-System (W)

$$W = 1 / (\mu - \lambda)$$

For the example with λ=4/hour and μ=5/hour,

W = 1/(5-4) = 1 hour.

3. Average Number of People-in-System (L)

$$L = \lambda W$$

For the example with λ=4/hour, μ=5/hour and $W = 1/(5-4) = 1$ hour,

$L = 4W = 4$ people in the system.

4. Average Wait Time (Wq)

$$Wq = W - 1/\mu$$

For the example with λ=4/hour, μ=5/hour and W = 1/(5-4) = 1 hour,

Wq = W - 1/5 = .80 hour wait.

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5. Average Line Length (Lq)

$$Lq = \lambda \, Wq$$

For the example with λ=4/hour, μ=5/hour and
Wq = 1 - 1/5 = .80 hour,

Lq = 4 Wq = 3.2 people waiting in line.

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